AD 296 979

ASD-TDR-62-924

A Contribution to the Theory of Schlieren Sensitivity and Quantitative Evaluation

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TECHNICAL DOCUMENTARY REPORT NO. ASD-TDR-62-924

December 1962

Directorate of Engineering Test
Air Force Systems Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

Project No. 1426, Task No. 142610

NOTICES

19-444, 400, 2-18-63

FOREWORD

This Technical Documentary Report was prepared in the Instrumentation Branch, Aerodynamics Division, Directorate of Engineering Test of the Aeronautical Systems Division as part of a general program of development and studies of gasdynamic measuring techniques. The author wishes to thank T. A. Lapenas of Optron Laboratory, Dayton, Ohio, for many stimulating discussions.

ABSTRACT

Detectors used to evaluate schlieren images have a response proportional either to the illumination or to the logarithm of the illumination. Sensitivity point functions appropriate to both kinds of response are defined analytically. These functions are well defined even in the presence of optical system imperfections and are not referenced to the clear field illumination. By the choice of an appropriate system configuration, either function can be reduced to a constant. It is shown that, in principle, the light deviation values required for quantitative evaluation can be found by subtracting out the illumination increments due to diffraction and other system errors.

A qualitative treatment of schlieren diffraction from the point of view of the Thomas Young theory is given. A technique is described for obtaining a stigmatic source image with an offset two-mirror schlieren system. A simple method for obtaining simultaneous vertical and horizontal knife-edge schlieren pictures is given. A means for magnifying sensitivity without the use of long focal length mirrors is described. A new method for measuring gas temperatures, which involves crossing a color schlieren system with a spectrograph, is described and illustrated with an example.

This report has been reviewed and is approved.

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a, b, c	constants
D	photographic density = $\log_{10}(1/T)$
e	base of natural logarithms
E	illumination in schlieren image, calculated on the basis of geometrical optics
E^*	increment to schlieren image illumination due to other causes
$\mathcal{E}$	total illumination in schlieren image = $E + E^*$
k	constant
log	natural logarithm
$\log_{10}$	decadic logarithm
n	refractive index
R	linear response range: range of source image deflections that can be measured at constant sensitivity (mm)
$\tilde{S}$	sensitivity function = dE/dx
S	sensitivity function = $d(\log E)/dx$ (mm^{-1})
$\mathcal{S}$	sensitivity function = $d(\log \mathcal{E})/dx$ (mm^{-1})
T	transmission
u	coordinate of source image (or source slit) normal to knife-edge
x	distance, normal to knife-edge, between knife-edge and a point fixed relative to source image (mm)
x_0	value of x when light is not deviated
$x-x_0$	source image deflection normal to knife-edge (mm)

LIST OF SYMBOLS (Continued)

γ slope of characteristic curve of photographic negative

Δ density of neutral filter = $\log_{10}(1/T)$

ϵ projected angle of light deviation

ξ schlieren image coordinate normal to knife-edge

η schlieren image coordinate parallel to knife-edge

GLOSSARY

(See Fig. 9)

light source. Effective light source: image of primary luminous source formed by condenser and diaphragmmed by source slit.

source slit. Diaphragm that defines the contour of the effective light source.

object field plane. Idealized plane of the disturbance under investigation.

source image. Image of source slit formed by light through a point of the object field plane.

composite source image. Image of source slit formed by full aperture of optical system.

schlieren stop. Knife-edge, graded filter, or other cut-off device.

schlieren image. Image of object field plane.

standard schliere. Weak negative (or positive) lens that produces known light deviations; in use is located at the object field plane and serves as a calibration reference in the schlieren image.

noise. Light deviations produced by air inhomogeneities in the schlieren beam outside of the disturbance under investigation.

I INTRODUCTION

The sensitivity of any physical instrument can be defined as the slope of the curve relating the instrument indication to the quantity measured. Linearity refers to the closeness with which the curve approaches a straight line (ref 1). The purpose of measurements with the schlieren method is to obtain information about such quantities as density gradients in gas flows, ruling errors in concave gratings, the quality of mirrors and transparent optics, etc. This information is derived from the quantity actually measured: the angle that light is deviated at points in the object field under investigation. The primary indication of the schlieren instrument is the illumination in the image of the object field. To measure the illumination one or more supplementary detecting and recording instruments (the eye, photographic film, photoelectric device, densitometer, photometer) must be introduced, each with its own sensitivity and linearity characteristics (refs 2,3). The response of these instruments is in general more or less linear with respect to either the illumination in the primary schlieren image or to the logarithm of this illumination. It is appropriate and useful in the consideration of schlieren sensitivity to account for both types of response.

In defining schlieren sensitivity it is convenient to use the distance x (in millimeters) between the knife-edge and a point fixed relative to the light source image formed by light through an object field point as the independent variable, i. e., the quantity measured. The distance is measured normal to the knife-edge. The value of x which would be indicated if the light were not deviated at the object field point is designated x_0. The quantity $(x-x_0)$ is the source image deflection corresponding to the projected angle of light deviation ϵ; $(x-x_0)$ and ϵ are taken positive when the deflection is away from the knife edge. From the system constants ϵ is known if $(x-x_0)$ is known. The total illumination $\mathcal{E}$ at a schlieren image point can be represented as the algebraic sum of two components: the direct illumination E computed on the basis of geometrical optics which is a function of x only, and a component E* contributed by such phenomena as interference, diffraction, and flare and stray light.

The component E* is often small compared to E and can be neglected. Significant sensitivity criteria can accordingly be defined in terms of both $\mathcal{E}$ and E. These criteria are the functions $\tilde{S} = dE/dx$, $S = d(\log E)/dx$, and $\mathcal{S} = d(\log \mathcal{E})/dx$. No use will be made of the fourth criterion, $d\mathcal{E}/dx$. The sensitivity criteria are defined at points in the schlieren image and are functions of x, hence in general vary from point to point as ϵ and x vary. Further, the x_0 values may also vary from point to point due to system imperfections, so that points with equal ϵ values are rendered at different sensitivities. It is apparent that the assignment of a single sensitivity number to a schlieren image or to a given system adjustment is misleading, unless the sensitivity function is constant with x.

The schlieren image is evaluated visually in most cases, either directly on a screen or through the medium of the photographic reproduction. To a good approximation through a restricted range of illuminations, visual sensation and photographic density are directly

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Manuscript released by the author 15 October 1962 for publication as an ASD Technical Documentary Report.

proportional to log illumination (ref 3). The most useful sensitivity criteria are accordingly the logarithmic functions S and $\mathcal{S}$. Schlieren sensitivity in this sense is conventionally defined as the proportional change in illumination relative to the undisturbed or clear field illumination per unit angular deviation of the light rays from their clear field direction. (E^* is neglected.) This is seen to be an approximate formula, valid in general only when the deviation is small, for the function S (to within a multiplicative constant) evaluated at a particular x_0. This definition, besides being inadequate and misleading, is not in a form suitable for further analytical development. Nevertheless, a conveniently simple relation between the standard formula and the sensitivity function S may be noted. In a conventional schlieren system configuration with rectangular source slit and straight knife edge, the value of S for small light deviations is numerically equal to the reciprocal of the knife edge setting in millimeters.

The visibility of detail in a schlieren image depends (subject to the vagaries of human vision) on the space rate of change of log illumination as a function of the space rate of change of x. We shall let ξ be the image coordinate normal to the knife edge and consider the special case where E^* is constant or at most a function of x only. We have

$$\frac{d(\log \mathcal{E})}{d\xi} = \frac{d(\log \mathcal{E})}{dx} \frac{dx}{d\xi} = \mathcal{S}\frac{dx}{d\xi}. \tag{1}$$

Subject to the restriction on E^*, we see that the sensitivity function $\mathcal{S}$ at any image point can be interpreted as the space rate of change of log $\mathcal{E}$ per unit space rate of change of x in the ξ direction. The visibility of any given item of detail $dx/d\xi$ varies from point to point in the image as $\mathcal{S}$, a function of x, varies. If $\mathcal{S}$ is constant with x, we should expect the visibility of such an item of detail to be sensibly constant regardless of the magnitude of the disturbance in which the detail is imbedded.

We shall consistently use the unmodified expression "source image" to mean the image of the source slit formed by light through a single point of the object field plane. When the image formed by the full aperture of the optical system is intended, the expression "composite source image" will be used. This distinction should be clearly understood. It should be realized that the individual source images are essentially sharp and aberration-free, because of the small effective aperture, even when the composite source image, formed by the full aperture, is badly distorted due to aberrations and inferior optics. When the light is deviated at a point in the object field, only the source image corresponding to this point is deflected. The knife-edge is the aperture stop of the optical system that images the object field. The sensitivity functions are defined in terms of the source images. Consequently, if the system response can be linearized with respect to the magnitude of the source image deflections (normal to the knife-edge), the sensitivity will not vary over the field whether or not the optical system is perfect.

In what follows we shall investigate the form of the sensitivity functions as related to the contour of the light source slit and the structure of the cut-off device, with emphasis on configurations giving constant sensitivity. A discussion of schlieren system errors, including diffraction, will lead up to the subject of quantitative evaluation of schlieren images. The treatment will be limited to the derivation of formulas for finding the light deviation values in the presence of system errors, since the evaluation procedure from this point on is well established. Measures for extending the sensitivity limits of the schlieren method will be investigated. An example will be given to illustrate the application of a quantitative schlieren technique to the measurement of gas temperatures.

II SCHLIEREN SENSITIVITY

When a straight knife-edge is used as the cut-off device, the form of the sensitivity functions $\tilde{S} = dE/dx$ and $S = d(\log E)/dx$ is determined by the contour of the light source slit. It is convenient to carry through the analysis in terms of the coordinates of the source image corresponding to a single arbitrary point in the object field plane. The source image is assumed to be geometrically similar to the effective light source. The source images corresponding to different field points are assumed to be equal in size, to be uniformly and equally illuminated by light from the source, but not to be necessarily coincident. If the source images are rectangular and the knife-edge is aligned parallel to one edge, the illumination is required to be uniform only in the direction normal to the knife-edge.

In figure 1, f(u) is the upper contour of the source image. It is convenient and permissible to use the u-axis as the lower contour $\left(\text{i.e., let } f_1(u) \text{ and } f_2(u)\right.$ be the actual upper and lower contours and define $\left.f(u) = f_1(u) - f_2(u)\right)$. Let x be measured from the point $u = 0$. Then the illumination component E at the schlieren image point conjugate to the object field point is $E = \text{constant} \cdot \int_{-\infty}^{x} f(u)du$. We have

$$\tilde{S} = \frac{dE}{dx} = \text{constant} \cdot f(x)\,, \tag{2}$$

$$S = \frac{d}{dx}(\log E) = \frac{dE/dx}{E} = \frac{f(x)}{\int_{-\infty}^{x} f(u)du}\,. \tag{3}$$

The condition for constant $\tilde{S}$ is $f(u) = \text{constant}$, i.e., a rectangular contour.

If $f(u) = bu^p$, $b = \text{constant}$, $p \geq 0$, $u > 0$; $f(u) = 0$, $u < 0$; then $S = (p + 1)/x$ for $x > 0$ and $S = 0$ for $x < 0$. An exponential contour gives a constant S, since if $f(u) = ae^{ku}$ where a and k are constants, then $S = k$. Functions f(u) can readily be written down for which S, after passing through a minimum, increases with x; but this case is of no interest. Contours of particular interest and importance include the rectangular and triangular $\left(p\right.$ equal to 0 and 1 respectively in $\left.f(u) = bu^p\right)$, the circular, and the exponential ($S = \text{constant}$). When a rectangular source is not aligned perfectly, it becomes a triangular source for at least a small range of values of x.

Log E is plotted against x in figure 2 for the rectangular, triangular, and unit-radius circular contours. The function S for any x is the curve slope. The curve for an exponential contour is of course a straight line with slope $S = k$. The curves are plotted so that the ordinates are arbitrarily made equal at $x = 0.3$ mm. In the D/γ ordinate scale in figure 2, D is the density of the photographic negative and γ is the slope of the characteristic curve of the negative. The ordinate scales are relative and are related by the formula $D/\gamma = (\log_{10} e)\log E + \text{constant}$. (The constant is set equal to zero in figure 2.)

It will be seen later that when the source is rectangular, the illumination component E^* is to a good approximation independent of x. We then have $d\,\mathcal{E}/dx = dE/dx = \tilde{S}$ = constant. Nothing is gained by knowing the precise functional form of $d\,\mathcal{E}/dx$ when it is not constant, consequently we have not assigned a separate symbol to this quantity.

The logarithmic sensitivity functions S and $\mathcal{S}$ are never equal unless E^* is zero. The effect of a positive E^* is to increase the ordinates of all of the curves in figure 2, by a negligible amount at large values of x and by increasingly larger amounts as x approaches zero; in the same way that flare light causes the characteristic curve of a photographic negative to depart from the sensitometric curve (ref 4). Later on, after we have examined the effects of diffraction and other system errors on E^*, we shall derive approximate expressions for the function $\mathcal{S}$. The curves in figure 2 retain their same positions relative to each other, with reduced slopes toward the left side of the figure. Fortunately, a satisfactory approximation to a curve with constant slope can still be obtained, by terminating the exponential contour $f(u) = ae^{ku}$ on the left at a suitable value of u. A light source slit with such a contour, made to a formula to be given later (equation 9), is shown in figure 3.

The form of the sensitivity functions is affected by the structure of the cut-off device as well as by the contour of the source slit. If the composite source image is rectangular and stigmatic, an exponentially curved or other knife-edge contour can be used instead of a straight knife-edge. The preceding analysis is not changed essentially by this inversion, although diffraction effects are more troublesome in practice.

A different technique, developed by Holder and North (ref 5), involves the use of a graded neutral density filter as the cut-off device. A point or line light source is used. We shall derive a formula for a linear filter to give constant sensitivity S. If Δ is the neutral density of the filter and T is the transmission, $\Delta = \log_{10}(1/T) = -0.434 \log T$. The illumination component E in the schlieren image is directly proportional to T, so the condition for constant S becomes $S = d(\log E)/dx = dT/(Tdx) = k$, or $\log T = kx$ + constant. Hence, $\Delta = \Delta_0 - 0.434\,kx$, where Δ_0 is the density at $x = 0$ and k is the desired constant value of S. Finally, $d\Delta/dx = -0.434\,k$. Linear graded filters can be made by photographing at various magnifications a standard schliere mounted in the parallel schlieren beam using an exponential source slit and straight knife edge.

An extremely fine grain, high contrast film or plate should be used. Nonlinear filters can be made using other source contours. (In a variation of the filter technique, a wedge interference filter or a color transparency of a grating spectrum can be used as the cut-off device to give an image in color.)

The exponential source and the linear graded filter techniques complement each other. We shall define the linear response range R to be the total range of source image deflections, in millimeters, which can be measured at constant sensitivity. For large ranges, the exponential source slit tends to become too wide for use with available high-brightness light sources. For small ranges, the necessarily finite width (of the order of one millimeter) of the line or point light source limits the application of variable transmission filters. The dividing line can be set at about R = 3 mm. For a given ratio between the maximum and minimum illuminations in the schlieren image, the (constant) sensitivity $\mathcal{S}$ and the linear response range R are inversely proportional,

$$\mathcal{S} \cdot R = \log\left(\mathcal{E}_{max}/\mathcal{E}_{min}\right) \tag{4}$$

In general terms, the graded filter technique measures a large range of deflections at low sensitivity; the exponential source technique measures a small range of deflections at high sensitivity. If coverage of a large range at high sensitivity is required, measurements with the exponential source technique can be made at a number of knife edge settings. The formula given above for linear filters provides a constant S rather than a constant $\mathcal{S}$, but the difference is not significant at the low sensitivities which obtain.

The constant sensitivity techniques have a number of advantages. A set of exponential slits and linear graded filters can be provided so that a single basic schlieren system can be used to measure a wide variety of phenomena under optimum sensitivity and range conditions. Change-over from one range to another is simple. If the linear response range is not exceeded, the knife edge or filter setting is not critical: measurements made at different times or under conditions of moderate vibration are directly comparable.

Another important advantage of the constant sensitivity techniques is that, by choice of a suitable range, approximately constant visibility of detail in the schlieren image is retained at all x-values, as was noted in the Introduction. Holder and North (ref 5) associate the improvement in the quality of schlieren photographs obtained with graded filters with an amelioration of the effects of diffraction at the knife edge. This is perhaps not the most significant factor, since a similar improvement in quality is observed in photographs obtained with the exponential-source knife-edge configuration and at higher sensitivities. Detail in disturbances causing large light deviations is normally not observed with the conventional rectangular-source knife-edge configuration, since the source images are deflected either entirely onto the knife blade causing a complete loss of detail, or away from the knife edge to large x-values where the slope of the rectangular source curve in figure 2 is small.

The uniform visibility of detail for a constant sensitivity system configuration can be retained even when there exist large system imperfections of certain types. Equation 1 can be rewritten as

$$d(\log \mathcal{E})/d\xi = \mathcal{S}\, dx/d\xi = \mathcal{S}\left[d(x-x_0)/d\xi + dx_0/d\xi\right]. \quad (5)$$

If the values assumed by $dx_0/d\xi$ are generally small compared to the $d(x-x_0)/d\xi$ values, we see that the visibility of any given item of detail $d(x-x_0)/d\xi$ will be approximately constant if $\mathcal{S}$ is constant. Even though the range of the x_0 values themselves may be quite large due to incorrect overall curvature of windows and mirrors, the $dx_0/d\xi$ values will usually be small if the polishing is smooth and of high quality without local discontinuities or abrupt changes of curvature.

Figures 4 and 5 show an extreme case where a very poor plastic window was mounted in the parallel beam of a two-mirror schlieren system. The knife-edge setting was 1/5 mm, i.e., S = 5, for the center of the field in figure 4. Thus the sensitivity function S had the same value in the center of the field in figure 4 as over the whole field in figure 5, yet the disturbance which was rendered in figure 5 with reasonably satisfactory detail could not be measured at all under the conditions of figure 4.

It must not be inferred from the preceding discussion that a general relaxation in the quality requirements for schlieren optical components is permissible with a constant sensitivity system. It is the total linear response range and the sensitivity that are inversely proportional; the optimum sensitivity with which a disturbance with a given range can be measured is reduced if an increment to the range must be added to account for system imperfections.

In some applications of the schlieren method it is absolutely necessary to use windows of poor optical quality, and reduced sensitivity must be tolerated. It is here that the constant sensitivity techniques can produce the most striking improvements over existing methods. Efforts have been made to adapt multiple-source schlieren (ref 6) to this situation in order to "focus out" the poor windows. This effort is self-defeating. When not one but a number of source images are effective in imaging each object point, the quality (i.e., absence of optical aberrations) of each source image should if anything be improved if the net effect is not to be deterioration of the imagery in the schlieren picture. Actually, the source images are of poor quality even with no windows. Furthermore, the light from an object point traverses different paths to the various effective source images, so that not only are system imperfections, caused by poor windows, different at different object points as in a conventional single-source system, but for each object point the error may be different for each effective source image. Finally, diffraction effects are greatly aggravated by multiple knife-edges or other multiple cut-off devices.

III SYSTEM ERRORS

Before undertaking a discussion of quantitative schlieren it is necessary first to investigate system errors, i.e., factors which produce unwanted effects in the schlieren image. These can be grouped roughly into two classes: factors which affect E and factors which contribute to the wholly unwanted component E*.

In the first group are included factors which cause the x_0 values to vary over the field: imperfect optical elements, faulty optical design, and imperfect adjustment and alignment of the system. These matters are discussed in the standard treatises on the schlieren method (such as refs 5 and 9). A requirement of flatness to within a wavelength or so for window surfaces, particularly for thin windows of large surface area, is unrealistic. In use there frequently exist temperature and pressure differences between the faces sufficient to cause a bending of many wavelengths. A small spherical lens power as well as prism power in windows is permissible and can be completely compensated for by small longitudinal and lateral knife-edge adjustments. Irregular gradients in total curvature and a "lemon peel" surface are damaging. The only satisfactory way to test schlieren windows for the net effect of total surface curvature, wedge angle, and refractive index variation is by the schlieren technique itself; consequently a reasonable way for the user to state requirements is in terms of the schlieren effects produced. Similar remarks apply to plane and concave schlieren mirrors.

The distribution of x_0 values is also affected by random error (noise), caused by air inhomogeneities in the schlieren beam outside the object field. This error can be anything from negligible to very large, depending upon the magnitude of the light deviations in the object field and the desired precision of the measurements. The error can be minimized by temperature control, by shielding the beam from light source to knife-edge, and in extreme cases by evacuation of the system.

The last source of error in this group to be considered is nonuniformity of brightness of the light source. The Ronchi grid technique of schlieren evaluation, which is of limited application, does not require uniformity of brightness, nor does the graded filter technique. Uniformity of brightness of a rectangular source is required only in a direction normal to the knife-edge; a capillary mercury arc source should be oriented so that the knife-edge is not parallel to the long dimension of the arc. (But see below.) Flash and spark sources and "compact" mercury and xenon arcs are not likely to be sufficiently uniform in brightness unless a special technique to be described is used. Defocusing the effective source slit with respect to the primary image of the source formed by the condenser lens, a technique which is satisfactory in quantitative spectroscopy, is not sufficient here. Not only must the source images corresponding to each point in the object field be uniformly illuminated, but they must also be equally illuminated, i.e., the object field must also be uniformly illuminated.

The following procedure can be applied only to a two-mirror, off-set, astigmatic system. Two slits at right angles are used at the effective source location, separated by the astigmatic difference in focus of the system, with the slit nearest the source oriented parallel to the system plane. (If the primary source is long and narrow, as is for example a capillary mercury arc or ribbon filament lamp, the slit parallel to the long dimension is located near the condenser-formed source image; if the primary source is approximately round, the condenser image is located roughly midway between the slits; etc.) An effectively anastigmatic rectangular composite source image is then formed at the knife-edge location, provided that the x_0 values do not vary significantly over the field. The maximum width of the slits is determined by the requirement that rays traced back through the system from every point of the composite source image must pass through both slits and intersect a luminous point of the primary source. Compliance with this requirement is guaranteed in practice if, when the knife-edge is advanced from each of the four diagonal 45-degree directions, the entire field remains illuminated until the light is completely cut off. The nonuniform darkening due to any variation of the x_0 values is readily distinguished from that due to excessive slit width. A little consideration shows that a good approximation to uniformity of illumination of both the object field and of the source images corresponding to object field points is obtained, even when the light source departs substantially from uniformity of brightness.

A number of supplementary benefits result from this stratagem. The knife-edge is not restricted to two orientations but may be inserted from any direction, and longitudinal adjustment is not required when changing orientations. This greatly eases and speeds adjustments, facilitates remote control and automatic operation of the system, and makes possible the investigation at maximum sensitivity of light deviation gradients of special interest. The composite source image is necessarily rectangular, but the straight knife-edge can be replaced by an exponentially contoured edge to give constant visual sensitivity (In this case there are only two permissible orientations). At low sensitivities the graded filter technique can be used. A straight knife-edge, and sets of exponentially contoured knife-edges, graded filters, and color filters, each with a range of sensitivities, can be provided to give maximum flexibility to the system, with virtually all adjustments made at the knife edge location.

The factors which contribute to the unwanted illumination component E^* include flare and stray light, diffraction and interference effects, and scattering. We shall restrict our definition of flare to include only light which is multiply reflected at the surfaces of windows and other optical elements in the schlieren beam, the effect of which is to form ghost images of the light source in the vicinity of the knife edge, and generally to illuminate the schlieren image uniformly. The entire area of the light source may contribute to flare, consequently the source slit should be no wider than is required to provide the needed range and to avoid interference effects to be discussed below. Flare can be eliminated by introducing a wedge angle between the surfaces of windows and other glass plates in the system, and by mounting such elements at an angle to each other. The angles need be only a few minutes, sufficient to deflect the ghost images onto the knife blade or other stops provided for this purpose.

Stray light is taken to be light which reaches the schlieren image from sources other than the schlieren light source. This error can be eliminated or minimized by removing the sources, shielding, introducing stops in the vicinity of the knife-edge, and minimizing the shutter-open time in flash photography. If the object field under investigation is a self-luminous gas, it may be desirable to use a high-intensity line source and a filter introduced behind the knife-edge to pass a narrow wavelength band.

IV DIFFRACTION

The remaining schlieren system errors to be discussed will be treated under the general heading of diffraction.

Schlieren effects are geometrical optical phenomena. When the wave nature of light intrudes and produces observable effects, these effects must be treated as errors to be eliminated if possible. We should like to treat diffraction in the schlieren system as a problem in geometrical optics. A theory which lends itself to such treatment is the Thomas Young theory (refs 7,8). We shall adapt the macroscopic aspects of this theory to our purpose as required to "explain" the observed phenomena.

According to our version of the Thomas Young theory, the contribution to the illumination of an image by light that does not graze the edges of diffracting objects en route is computed as predicted by geometrical optics. Light rays which graze the edges of diffracting objects experience a kind of reflection (or "deflection" and "inflection") and fan out in planes which we shall call diffraction planes. The orientation of the diffraction plane for any ray is determined by two lines: the line of travel of the ray as it impinges on the edge, and the normal to this line and the edge at the point of intersection. The diffracted light is again treated by geometrical optics. The total illumination at an image point consists of the direct light plus edge contributions associated with all the diffraction planes which intersect the image point.

As noted in the Introduction, we use the expression "source image" to mean the image of the source slit formed by light through a single point in the object field plane. From this point of view the classical Fraunhofer diffraction pattern at the source image plane is formed by the superposition of all the source images plus out-of-focus edge contributions from the diffracting aperture edges. We do not observe this pattern and are not concerned with it.

We shall first consider the effect on the schlieren image of diffraction at a straight knife-edge. Rays emanating from an object field point which strike the blade are stopped. Rays clearing the blade are brought to a focus at the conjugate image point. Grazing rays are diffracted in diffraction planes which intersect in a line in the image plane passing through the image point. We shall call this line the diffraction line for the image point. The diffraction lines for all image points are parallel. The illumination at any image point, therefore, consists of the direct light from the object field point plus edge contributions from light originally directed toward all illuminated points on the diffraction line for the image point. This increment to the direct light illumination can be considered to be the sum of contributions from infinitely many lines of light in the source images that graze the knife-edge, each of zero width.

The sharply demarcated dark shadows which can sometimes be observed extending out into the illuminated field from opaque objects, when a sensitive knife edge adjustment has cut off most of the direct light, are simply explained: the illuminated lengths of the diffraction lines which determine the magnitude of the edge contributions to the shadowed areas are shortened by the presence of the opaque objects. Similarly, the images of opaque objects are illuminated in the direction of any diffraction lines from directly illuminated areas of the field that traverse them.

For a uniformly bright rectangular light source, if the range is not exceeded, the illumination at any point from light diffracted at the knife-edge only is independent of the x-values along the diffraction line in the image plane, since the length of the line of light that grazes the knife-edge is constant with x.

We shall next consider diffraction in the object field. Light that does not graze the edges of opaque objects in the object field is treated according to geometrical optics until it reaches the vicinity of the knife-edge where the preceding discussion applies. Light that does graze the edges of opaque objects in the object field fans out in diffraction planes oriented as specified above. This diffracted light is again treated according to geometrical optics and, in the absence of a knife-edge or other stops in the system, is brought to a focus in the schlieren image and no effect is observed. With the knife-edge present, part of the diffracted light is cut off, part proceeds on to form the image of the edge, and the grazing part is again diffracted. The effects observed depend upon the orientation of the diffraction planes and the degree of cut-off of the direct light. The halo around the edges of opaque objects in the field receives a simple explanation in the Thomas Young theory. As the knife-edge is advanced and the direct light is cut off, the light from object field edges diffracted in diffraction planes normal to the knife-edge is cut off only very slowly; more rapidly as the planes approach parallelism with the knife edge. The halo remains as the knife-edge is advanced some distance past the point where the direct light is completely cut off. If the diffraction planes are parallel to the knife-edge, the diffracted light is cut off in the same degree as the direct light from points adjacent to the edge and no halo is observed.

The light that is diffracted at the edges of opaque objects in the object field, and again at the knife-edge adds another increment to the illumination in the schlieren image. The increase is confined mostly to the immediate vicinity of the edge, is most intense at the edge, and falls off rapidly in both directions away from the edge. (The shadow phenomenon described earlier is counteracted in the illuminated field near the edges of opaque objects by this added illumination.) Consideration of the diffraction planes into which the light is diffracted both in the object field and at the knife edge shows that the increment is greatest when the object field edge is parallel to the knife edge and drops to zero for an edge normal to the knife-edge. The increment can be considerable when there are a number of edges in the object field close together and more or less parallel to each other and to the knife edge. This knife-edge orientation is to be avoided. The entire area of the effective light source contributes to the light diffracted at object field edges, as with flare light. For the light that is not again diffracted at the knife-edge this only increases the intensity of the halo and does no harm. The magnitude of the increment of illumination added by the second diffraction at the knife-edge is however directly proportional to the source area. The importance of minimizing the source width is again shown. For the rectangular source case the change in log $\mathcal{E}$ due to this doubly diffracted light is not affected by decreasing the source length (i.e., dimension parallel to knife edge). Optical aberration considerations, which have so far been neglected, dictate that the source length should also be minimized.

Light diffracted at the edges of opaque objects at other locations in the schlieren beam is treated in the same way as at object field edges, except that here the once diffracted light (to which the whole area of the source contributes) which gets past the knife-edge and other stops is not brought to a focus and an area rather than a line in the schlieren image is illuminated. The area affected depends on how much the diffracting edge is out of focus. These edges, e.g., the rims of concave mirrors, should be eliminated by locating the field stop of the schlieren image optical system at or near the object field plane, if necessary by installing a stop for this sole purpose.

We shall consider scattered light to be light that is deviated by dust particles, scratches, bubbles, striae, etc., on mirror surfaces and within or on the surfaces of all transparent optical elements between the luminous area of the primary light source and the schlieren image. The effect of scattering ahead of the knife-edge is similar or identical to the effect due to diffraction at the edges of opaque objects not in the object field. This error (the whole area of the source again contributing) can be very disturbing and can only be eliminated by eliminating the scattering sources. If a lens is used after the knife-edge, it should in most cases be a single-element simple lens or cemented achromat, kept clean and free of scratches (and coated to reduce flare).

We shall now consider diffraction at the light source slit. Light that grazes the edges fans out in diffraction planes with the usual orientation. Points in the object field are illuminated by direct light plus edge contributions associated with all diffraction planes intercepted. We consider that the uniformity of illumination of the object field is not significantly affected, and that the previous discussion of diffraction at object field edges and at the knife-edge is applicable to the total light. It follows that the only significant effect of diffraction at the source slit is to cause a lack of uniformity of illumination of the edges of the source images, due to imperfect sharpness of the source slit edges and the finite aperture of the condenser system that images the primary source at the slit.

An exception to the preceding analysis must be made when the source slit is very narrow in a direction parallel to the knife edge, when a significant fraction of the total illumination at object field points is due to diffracted light. With the knife-edge or other stop in position, interference fringes superimposed on schlieren effects are observed under certain conditions in the schlieren image. The effects are most pronounced when the slit width is about 0.03 mm or less, but are sometimes observed with reduced intensity when the width of the slit drops to a few tenths of a millimeter anywhere along its length. Since the schlieren image is degraded by the appearance of discrete fringes, it may be desirable for safety to use a figure of at least one millimeter as the minimum source slit width. A number of writers have suggested that positive use be made of the fringe patterns that can be produced in a schlieren system, for the purpose of making quantitative measurements. Unfortunately, interpretation is difficult, and the attainable sensitivity incident to the fringe displacements is much less than is the true schlieren sensitivity.

The aperture of the ray pencil that forms the image of an object field point (after traversing the non-cut-off portion of the corresponding source image) is everywhere very small when the source slit dimensions are small. Consequently, the effect of optical aberrations on the source images and on the schlieren image is generally negligible in a properly designed mirror schlieren system (except possibly for distortion in the schlieren image). The aberrational error is increased as the source slit length parallel to the knife-edge is increased. The net effect on the source images of imperfections in the optical elements, residual aberrations, and diffraction at the source slit is to cause a lack of sharpness and uniformity of illumination of the edges.

The increments to the schlieren image illumination due to diffraction at edges other than the knife-edge are seen to be independent of x for all light source contours.

The deleterious effects due to diffraction are greatly aggravated when a second edge, or a number of edges, are placed at the knife-edge location. The analysis could readily be extended to account for most of the observed effects. Schlieren configurations utilizing multiple knife edges, a narrow slit, or a grid with narrow slit spacings as the cut-off device are to be avoided. Some color transparencies used for color schlieren have a number of such effective edges at the boundaries between colors, apparently due to the properties of the color film. Stops placed in the vicinity of the knife-edge to intercept stray and flare light must be sufficiently displaced, depending on the geometry of the system, that they do not intercept or diffract a significant amount of the light once diffracted at object field edges. The treatment of diffraction at a straight knife-edge as given above does not apply without modification to an exponentially curved edge. A narrow tip on such a curved edge must be avoided.

Schardin (ref 9) has applied Fraunhofer theory to the light in the transparent regions of the object field and predicts that the minimum light deviation angle which can be detected in a given schlieren system is inversely proportional to the linear size of the disturbance producing the deviation. The prediction applies to visual discrimination of illumination differences for the particular case of a rectangular source. The same reasoning would predict a loss of definition in the schlieren image, as evidenced by a lack of sharpness of the edges of opaque objects, even if the knife-edge were completely removed. This effect is not observed. The theory is difficult to test experimentally, since isolating a field area by surrounding it with a diffracting edge vitiates the experiment. We have preferred to consider that the light is not diffracted in transparent regions of the field.

The theory of schlieren diffraction has been given by Linfoot (ref 10) for the case of the Foucault knife-edge test of a spherical mirror with a coherent point source, and by Speak and Walters (ref 11) for the case of an infinitely narrow coherent slit source and a rectangular field aperture. The calculated distribution of illumination in the schlieren image is in qualitative agreement with our findings: the intensity of illumination is increased near object field edges not normal to the knife-edge; opaque areas are illuminated; a halo appears, and disappears at edges normal to the knife-edge; etc. Speak and Walters with Schardin predict that the (visual) detectability of a given light deviation angle is a function of the linear size of the disturbance producing the deviation, but the practical significance of this prediction is uncertain because of the restrictive assumptions.

It is not possible to distinguish between light diffracted at an edge and light which undergoes ordinary reflection at the edge. The increase in the intensity of the "diffracted" light that occurs when a sharp knife-edge is replaced by a less sharp edge is readily observed experimentally. We have seen that the schlieren method requires an extended source if interference effects are to be certainly avoided. An extended source is normally required in any case to provide the necessary range. It would be of doubtful value to attempt a quantitative theoretical analysis of the schlieren diffraction image for the general case of an extended noncoherent source and for other than simple circular or rectangular object field contours. Fortunately it is also not necessary for our purposes. We shall investigate in the next section the conditions under which it is legitimate to "subtract out" the effects of diffraction and other system errors by the technique of evaluating a tare image.

V QUANTITATIVE SCHLIEREN

The theoretical basis for the quantitative evaluation of schlieren measurements is well understood (refs 9, 12, 13). Evaluation is possible in many cases of interest, provided that the ϵ values are known. The treatment here will be limited to an investigation of techniques for finding the ϵ values. It will be shown that this is practicable in general only if the system response is linear, i.e., if $\tilde{S}$, or $\mathcal{S}$, is constant.

Let the coordinates in the schlieren image be ξ and η, with the ξ - direction normal to the knife-edge. Quantitative evaluation is accomplished by numerical integration along a line η = constant, starting at a point $\xi = \xi_1$ where the quantity being evaluated is known. We have, at a point (ξ, x) in the ξ - x plane,

$$\mathcal{E} = E(x) + E^*,$$

where E^* is in general a function of x, ξ, and the x-values at all points on the line η = constant in the ξ - η plane. Then

$$\begin{aligned} d\mathcal{E} &= (dE/dx)\,dx + dE^* = \tilde{S}\,dx + dE^* \\ &= \tilde{S}\left[d(x-x_0) + dx_0 \right] + dE^*; \\ \tilde{S}\,d(x-x_0) &= d\mathcal{E} - (\tilde{S}\,dx_0 + dE^*). \end{aligned}$$

Nothing can be done with this equation in the general case. However, if $\tilde{S}$ is a constant for all x and ξ, and if E^* is a function of ξ only, then the quantity in parentheses on the right is an exact differential and we can write

$$\tilde{S}\,d(x-x_0) = d\mathcal{E} - d\mathcal{E}'.$$

We have seen that the conditions are satisfied when the light source is rectangular. Consequently we have ($\tilde{S}$ constant)

$$\tilde{S}\left[(x-x_0)_2 - (x-x_0)_1 \right] = (\mathcal{E}_2 - \mathcal{E}_1) - (\mathcal{E}_2' - \mathcal{E}_1'), \tag{6}$$

where the subscripts 1 and 2 refer to conditions at ξ_1 and ξ_2 respectively. The quantity $(\mathcal{E}_2 - \mathcal{E}_1)$ is obtained from measurements of the illumination in the schlieren image. If $x = x_0$ everywhere, i.e., if there is no disturbance in the object field, we see that $(\mathcal{E}_2' - \mathcal{E}_1')$ is obtained from measurements in a tare image. If a uniform wedge standard schliere producing a known light deviation is located in the object field, the quantity in brackets on the left is known and $\tilde{S}$ can be found. (Correction must be made for light absorbed and reflected by the standard schliere; the subscripts 1 and 2 refer to measurements at the same field point without and with the standard schliere in position; and the second term on the right in equation 6 is omitted).

The accuracy and precision of the measurements depend on the magnitude of the random errors, the accuracy and precision of the measurements of $\mathcal{E}$, and the degree to which the source images are uniformly and equally illuminated. Even when the effective light source is uniformly bright, we have seen that the source image edges are not perfectly sharp or uniformly illuminated; consequently, the adjustment should be made to exclude x-values near zero or the maximum. This effect has been observed experimentally in quantitative schlieren measurements by Holder and North (ref 14), and is well illustrated in their figure 3.

Other system errors can be substantial, provided that the errors and the system conditions are the same in both test and tare images. For example, the same lamp operated at the same voltage should be used, or if a window in the beam is opened and closed the ghost images should fall either on or off stops in both cases. In the discussion of errors that are not functions of x for a rectangular source, it was tacitly assumed that the knife-edge itself was fixed. However, a small variation in knife-edge settings should have negligible effect, and the evaluation equation 6 does not require the same setting for test and tare images.

It would appear that a photoelectric detector capable of scanning a schlieren image and recording the information, used with a high speed computer programmed not only to find the ϵ values but also to carry through the numerical integrations required for a complete evaluation, might be a practical development. With sufficiently rapid scanning it might be feasible to investigate non-stationary phenomena quantitatively; one tare image could serve for an indefinite number of test images. The information is in immediately usable form and does not require an intermediate operation such as the measurement of fringe shifts in an interferogram. Other quantitative testing methods utilizing emission and absorption phenomena are generally much more difficult and applicable only to stationary situations. It has previously been assumed that diffraction errors and imperfect optics precluded the attainment of the accuracy and precision required to justify schlieren evaluation.

We shall next consider the case where the detecting instrument gives an indication proportional to log illumination. The component E is directly proportional to the area of the source image not cut off by the knife-edge. The component E^* can also be set proportional to a source image area with the same proportionality constant. Since the constant drops out we can represent E and E^* simply as areas. We shall consider only the case where E^* can be taken to be constant to a sufficiently good approximation. The approximation will generally not be good immediately adjacent to the edges of opaque objects in the object field that are not normal to the knife-edge. We have (E^* constant)

$$\mathcal{E} = E(x) + E^*;$$

$$\frac{d(\log \mathcal{E})}{dx} = \frac{dE/dx}{E + E^*} = \frac{d(\log E)}{dx}\,\frac{E}{E + E^*}; \tag{7}$$

$$\mathcal{S} = S\,\frac{E}{E + E^*} = S\left[1 - \frac{E^*}{E + E^*}\right].$$

We see that the sensitivity functions S and $\mathcal{S}$ approach equality when E is large compared to E^*, i.e., when x is large.

For a rectangular source image, with b the length in mm parallel to the knife-edge, we have $S = 1/x$, $E = bx$, and we can set $E^* = bc$, where c is in mm and very small. Then

$$\mathcal{S} = \frac{1}{x + c}.$$

For a triangular source image with slope m, we have $S = 2/x$ and $E = mx^2/2$. The illumination increments due to diffraction at the knife-edge now depend on the lengths of the lines of light that graze the knife-edge. Since we are assuming E^* to be constant we must select some average knife-edge position x_0 and set $E^* = x_0mc$. We then have

$$\mathcal{S} = \frac{2}{x + 2cx_0/x}.$$

We see that $\mathcal{S}$ approaches S for x large, in both cases. In the triangular case, $\mathcal{S}$ is given approximately by $2/(x + 2c)$ when x is near x_0. As x tends to zero, the sensitivity in the rectangular case approaches a maximum given by the reciprocal of c, and in the triangular case approaches zero for all $x_0 > 0$. The formulas do not take into account the lack of sharpness and uniformity of illumination of the source image edges, or the nonlinear response of the eye and photographic film to log illumination at low illumination levels.

Because of the properties of the logarithmic function, quantitative evaluation is not feasible unless both E^* and $\mathcal{S} = d(\log \mathcal{E})/dx$ are constant. We then have, with E^* and $\mathcal{S}$ constant,

$$d(\log \mathcal{E}) = \frac{d(\log \mathcal{E})}{dx}\,dx = \mathcal{S}\,dx = \mathcal{S}\left[d(x - x_0) + dx_0\right];$$

$$\mathcal{S}\,d(x - x_0) = d(\log \mathcal{E}) - \mathcal{S}\,dx_0 = d(\log \mathcal{E}) - d(\log \mathcal{E}'); \tag{8}$$

$$\mathcal{S}\left[(x - x_0)_2 - (x - x_0)_1\right] = \left[(\log \mathcal{E})_2 - (\log \mathcal{E})_1\right] - \left[(\log \mathcal{E}')_2 - (\log \mathcal{E}')_1\right];$$

where as before the subscripts 1 and 2 refer to conditions at ξ_1 and ξ_2 respectively and, on the right, the first term is evaluated from a test image and the second term from a tare image.

We now undertake to find a light source contour that will give a known constant $\mathcal{S}$. We set $E = \int_{u_1}^{x} f(u)\,du$. We again assume that E^* is constant and can be represented by the product of some average dimension of the source image intersected by the knife-edge, $f(x_0)$, and a small transverse dimension c. We have

$$\mathcal{E} = E + E^* = \int_{u_1}^{x} f(u)\,du + cf(x_0);$$

$$\mathcal{S} = \frac{d(\log \mathcal{E})}{dx} = \frac{f(x)}{\int_{u_1}^{x} f(u)\,du + cf(x_0)} = k;$$

$$f(x) = k\int_{u_1}^{x} f(u)\,du + kcf(x_0).$$

This equation has the solution $f(x) = ae^{kx}$ with $u_1 = (\log kc + kx_0)/k$. The required contour is again the exponential $f(u) = ae^{ku}$, with the area under the curve to the left of u_1 deleted to account for the constant $E^* = cae^{kx_0}$.

When $kc = 1$, $u_1 = x_0$, and again a limiting value for the sensitivity is found to be $\mathcal{S} = k = 1/c$, the same as in the rectangular source case. The value of the dimension c will in any particular schlieren system depend on the sum total of the contributions to E^*. If all contributions to E^* except diffraction at the knife-edge are neglected, the infinitely many lines of light of zero width in the source images that contribute to E^* at a schlieren image point are effectively replaced by a single line of length $f(x_0)$ and finite width c. We have not attempted to present an analysis by means of which a value of c can be computed.

We have shown that diffraction irremediably sets a limit to the magnitude of the sensitivity function $\mathscr{S}$. There is no such limit in the case of the function $\tilde{S}$.

In order to construct a formula for exponential source slits, a value for c must be assumed. A formula that has been found to be satisfactory assigns to kx_0 a value of 4. A convenient value to assume for c is 0.018 mm, since then the expression for u_1 becomes simply $u_1 = (\log k)/k$. These slits will at least give a better approximation to constant $\mathscr{S}$ than if the entire area under the exponential curve were included, since it is hardly likely that the assumed value for c will ever be excessively high. The magnification of the source image is assumed to be unity. The formula follows:

$$f(u) = \pm\, ae^{ku}, \quad u_1 < u < u_2\,;$$

$$f(u) = \pm\, \tfrac{1}{2}L, \quad u_2 < u < u_2 + 1.0\,; \tag{9}$$

$$u_1 = (\log k)/k, \quad ku_2 = 6, \quad a = \tfrac{1}{2}Le^{-6}\,.$$

All distances are in mm and the slit length is L. The design knife-edge setting is $x_0 = 2u_2/3$, whence $kx_0 = 4$; where x_0 is measured from the point $u = 0$, not $u = u_1$. The contour is made rectangular for a distance of 1.0 mm beyond the end of the exponential contour to minimize interference effects in the schlieren image. The linear response range in mm is given by $R = u_2 - u_1$. The slit shown in figure 3 had $\mathscr{S} = k = 5$ and $L = 6$ mm, and was made from a formula which assigned a slightly different value to u_1 than the above.

This same formula, slightly modified, can be used for the exponential contour of knife-edges to give constant $\mathscr{S}$ with a rectangular stigmatic composite source image. At the smaller k values the narrow tip on the left is objectionable due to diffraction. The tip is widened by moving the left terminus of the linear response range, u_1, to the right and substituting an equal rectangular area to the left of u_1. The minimum tolerable width of the tip can be found by experiment, and should not be less than 0.5 mm.

Since it has been necessary to simplify the problem in order to derive a suitable evaluation equation involving $\mathscr{S}$, it is not to be expected that the accuracy will be as high as with the equation involving $\tilde{S}$, particularly at high sensitivities and in the immediate vicinity of opaque edges that are not normal to the knife-edge. Experience in the application of both techniques is required in order to assess their relative advantages.

VI HIGH SENSITIVITY SCHLIEREN

In some applications of the schlieren method, measurements must be made of phenomena producing very small light deviations. In these circumstances a complete quantitative evaluation is not often possible. Here, and indeed in general, information which is quantitative in a geometrical sense, and which serves to define the state in the geometrically delineated regions of the object field, is frequently the only information obtainable. Conditions for realizing the full potential sensitivity of the schlieren method will be examined.

The primary requirement is a basic system of high quality and state of adjustment, with noise minimized. The discussion of system errors in preceding sections included suggested means for correction. We shall assume the arbitrary rule that the range of source image deflections due to optical imperfections and noise should not exceed the range due to object field disturbances. We again emphasize that it is the magnitude of the light deviation gradients due to these causes which determines the visibility of detail within the range of a constant sensitivity system.

No information is known to the writer concerning the minimum gradient in log illumination that is detectable visually. The minimum detectable difference in log illumination values in contiguous uniformly illuminated areas is about 0.018. By the photographic process this difference can be reduced to 0.002 in the primary schlieren image (ref 15). The minimum detectable difference in source image deflections is given by the applicable log illumination difference divided by $\mathcal{S}$. In any particular schlieren system this value is divided by an appropriate length to give the minimum detectable angular light deviation.

As an example, consider a two-mirror single-pass system with the focal length of the second mirror equal to 5.15 meters, operated at a constant $\mathcal{S}$ equal to 20. The total linear response range calculated from equation 9 is R = 0.15 mm source image deflection or six seconds light deviation. The permissible system error range including noise is then three seconds. The ratio between the maximum and minimum schlieren image illuminations in the linear response range is $\exp(\mathcal{S}R) = e^3 = 20$, from equation 4. The minimum detectable light deviation difference is 0.036 seconds if the schlieren image is viewed directly, or 0.004 seconds if a suitable photographic technique is used.

It is sometimes possible to obtain an increased schlieren effect by suitable alignment of the disturbance in the system. For instance, when a light ray completely traverses a two-dimensional disturbance in a gas, from a region of constant refractive index n_1 to a region of constant index $n_2 > n_1$, it follows from Snell's Law that $n_2 - n_1 \simeq \epsilon_1^2/2 + \epsilon_1 \tan\theta$, where θ is the complement to the angle of incidence and ϵ_1 is the total angle that the light is deviated. The first term on the right is included only to show the value of ϵ_1 when $\theta \to 0$. It is seen that ϵ_1 and θ are nearly inversely proportional when θ is small, and when possible the alignment should be made to minimize θ. If we take $\theta = 5°$, and $\epsilon_1 =$ 0.004 seconds from the example above, $n_2 - n_1 = 1.7 \cdot 10^{-9}$. This is approximately the refractive index difference across a normal shock wave in air at a Mach number of five when the local temperature and pressure ahead of the wave are 250°K and one micron mercury respectively.

A marked increase in visual contrast sensitivity with reduced definition can be obtained by focusing the schlieren image directly on the sensitive element of an image converter such as a television camera.

Sensitivity $\left[\text{in the sense of } d(\log \mathcal{E})/d\epsilon\right]$ can be magnified by magnifying the composite source image with a highly corrected lens. The effect is the same as that obtained by increasing the focal length of the second mirror in a two-mirror system. The aperture of the lens should be adequately large so that light diffracted in the object field will not be significantly intercepted by the lens barrel, otherwise the results are unsatisfactory.

When the detecting instrument measures $\mathcal{E}$ instead of log $\mathcal{E}$, we have seen that the schlieren sensitivity is not limited by diffraction. The limiting factors are then the brightness of available light sources and the sensitivity of the detecting instrument.

VII SUMMARY AND DISCUSSION

This work began as an attempt at developing techniques for improving the quality of schlieren photographs at high sensitivities. The stimulus was the work of Holder and North (ref 5), who replaced the schlieren knife-edge with a neutral density linear graded filter and succeeded in improving the sharpness, although only at low sensitivities. The explanation for the improvement was not quite clear; it could not be assumed that a photograph with knife-edge would necessarily be inferior to a photograph with filter, if both were taken at the same sensitivity. As we have seen, the sensitivity in a conventional schlieren picture is not a constant but may range from zero to a high value.

As the subject of schlieren sensitivity was investigated further, it became clear that the concept needed to be clarified if progress were to be made. The standard treatises on the schlieren method define sensitivity in terms of the visual contrast between the illumination in the disturbed and in the undisturbed regions of the schlieren image. That something is wrong is shown by the fact that the standard formula predicts a constant sensitivity in a conventional schlieren picture, which is clearly false. A major part of this work has been devoted to a redefinition of the fundamental concepts, and to an investigation of some of the important relations which can be derived once the sensitivity functions have been put into proper analytical form.

From the beginning, a clear distinction has been made between the case where the instrument response is linear with respect to the angle of light deviation, and the case where the response is logarithmic. Suitable sensitivity point functions have been defined for each case. Since schlieren system configurations vary greatly, the functions have been expressed in terms of source image deflections rather than light deviation angles. In any particular system, conversion can be made by multiplying by a suitable constant, usually a focal length. The functions are well defined even in the presence of imperfections or misalignments of the optical system.

The effects of modifying the contour of the light source slit and the structure of the cutoff device have been investigated. When a photoelectric detector is used to directly measure the illumination in the schlieren image, the preferred system configuration has been found to be the conventional rectangular slit and straight knife-edge. When the schlieren picture is evaluated visually or by measuring the density of a photographic negative, it has been shown that constant sensitivity can be obtained with a configuration utilizing an exponentially contoured source slit at high sensitivities, or by applying the graded linear filter technique at low sensitivities. The improvement in the quality of schlieren photographs at high sensitivities is attributed to the fact that detail is brought out which is normally lost with a rectangular source slit. The expression for the visual sensitivity for a conventional schlieren configuration is found to be similar in form to the standard formula, so there is not a complete break with tradition. The rather obvious point may be made that sensitivity and range can correctly be said to be inversely proportional only if the sensitivities considered are constant.

The second major purpose of this work has been to investigate the conditions under which quantitative schlieren measurements may be feasible. The errors which must be minimized or accounted for have been investigated in some detail. A number of measures for minimizing error which have not been fully discussed elsewhere are suggested.

The source of error which has been treated least satisfactorily in the literature is diffraction. The usual procedure is to develop the theory of the schlieren method on the basis of geometrical optics, add the qualification that in practice the usefulness of the method is limited by diffraction, and describe some of the effects. When the development is rigorous and quantitative, it turns out to be highly mathematical, highly restrictive in the assumptions that have to be made (point or line coherent light source, simple circular or rectangular object field contour), and extremely difficult to interpret and apply to an actual functioning schlieren system. One of the conclusions of this work has been that the success of the schlieren method requires an extended noncoherent source. Even though the object field may be bounded by a simple geometrical contour, the effects under investigation occur in local irregular regions and the application of the theory to such regions is not obvious. It appears to be out of the question to construct a quantitative diffraction theory that can be applied a priori to a practical schlieren system.

One of the troublesome features of the theoretical treatments that make interpretation difficult is the primary emphasis that is placed on the diffraction image of the light source that is formed by the full aperture of the optical system. It is sometimes asserted that the ultimate sensitivity attainable with a given schlieren system is that which would be obtained with a strictly coherent point source. This does not appear to have any connection with reality.

The approach that has been followed in this work has been to present a simplified non-mathematical theory of diffraction which is qualitatively understandable and which predicts the major effects which are observed in the schlieren image. The diffraction image of the light source formed by the full aperture of the optical system has been considered only incidentally. Some quantitative relations have been developed, but they can be evaluated only experimentally in a given schlieren system.

Formulas have been derived which show explicitly how diffraction irremediably sets an upper limit on the attainable visual sensitivity. It has been shown that the sensitivity is not limited by diffraction when the schlieren image is evaluated photoelectrically.

Provided that the appropriate sensitivity function is a constant, it has been shown that it is possible in principle to subtract out the effects of diffraction and other system errors and to perform a quantitative evaluation of a schlieren image. The necessary evaluation equations have been derived and the procedure described. No application of these equations has been made, consequently their usefulness has not been established.

A number of techniques and procedures for improving schlieren performance, which are more or less incidental to the principal topics covered, have been discussed. These are summarized here.

(1) A technique for manufacturing graded linear filters has been given, which does not require elaborate darkroom equipment. Filters made by this method have been measured with a microdensitometer and found to have good linearity. A formula has been given for quantitatively measuring the sensitivity of the filters, with the aid of a densitometer.

(2) By use of the constant sensitivity techniques, it has been shown that satisfactory schlieren pictures can be obtained with inferior windows and optics, although at reduced sensitivity.

(3) A simple technique has been described for obtaining a stigmatic composite source image with an off-set two-mirror schlieren system.

(4) Measures have been suggested for minimizing flare in the schlieren image caused by multiple reflections in the windows and other optics. Specifications for schlieren windows frequently require that the surfaces be parallel to a close tolerance. This is undesirable.

(5) The undesirability of using a narrow slit or grid as a schlieren stop has been emphasized.

(6) It has been pointed out that a dull knife-edge increases the diffraction error. The effect of a nonmetallic edge has not been investigated.

(7) A formula has been given for the design of exponential source slits of any desired sensitivity and size.

(8) A sample calculation has been carried through from which can be deduced the procedure for choosing appropriate system parameters required to measure a disturbance producing a given estimated magnitude and range of light deviations.

(9) It has been pointed out that visual sensitivity can be magnified by magnifying the composite source image with a lens. This makes possible the design of a satisfactory system when space limitations preclude the use of long focal length mirrors. Alternatively, in an existing system, the usefulness of graded filters can be extended to higher sensitivities.

In conclusion, a few new techniques and potential applications of the schlieren method, not previously considered in this report, will be discussed.

A simple method for obtaining simultaneous vertical and horizontal knife-edge schlieren pictures has been found to work very well. A small wedge mirror, coated for partial reflection on both surfaces (approximately 20 percent on first surface and 30 percent on second surface), is inserted into the converging light beam just ahead of the knife-edge. It can be shown that the required wedge angle of the mirror is given approximately by $W/2nF$ radians, where W is the width of the object field, n is the refractive index of the glass, and F is the schlieren mirror focal length. Two composite source images are formed, two knife-edges are used, and a single lens focuses the schlieren images on the camera. The beam that completely traverses the wedge mirror is also available and can be used for a viewing screen, color schlieren picture, focused shadowgraph picture, motion picture camera, television camera, etc. Strategically placed stops are used to intercept the surplus source images formed by multiple reflections in the wedge mirror.

A potential application of quantitative schlieren is in the investigation of rarefied plasmas. One notes that both the refractivity and the dispersion per particle for electrons are much greater than for neutral atoms or ions (ref 16). In principle, local electron densities in axisymmetric flow fields could be measured by making simultaneous schlieren observations at two widely separated wavelengths, provided of course that the electron concentrations were great enough to produce measurable effects. The schlieren method is particularly advantageous here (ref 13). Other methods: spectrometry, optical and far-infrared interferometry, and microwave techniques, are generally restricted to the measurement of average density values in such cases.

The discussion of quantitative evaluation in this report has been confined to the investigation of methods for measuring the ϵ values. Coefficients for the numerical integration of the evaluation equation for axisymmetric phenomena have been given in a separate report (ref 13).

Techniques which directly or indirectly measure the magnitude of the source image deflections provide a simpler means of quantitative evaluation than the methods that have been discussed in this report. In most applications the deflections are too small to measure accurately. A method for circumventing this restriction will now be described.

When the source images are magnified by the technique described previously, the deflections are also magnified. A wedge interference filter can be used as the schlieren stop and a cross section of the region under investigation can be focused on the slit of a stigmatic, large aperture, low dispersion spectrograph. Evaluation of such a spectrogram is not difficult. Calibration can be made most easily with the aid of a standard schliere. Since diffraction effects do not affect the magnitude of the source image deflections, the accuracy should be good. Commercially available wedge interference filters are long and narrow and not well adapted to this technique. The filter should be approximately square. Since visual evaluation is not involved, the filter wavelength range could be extended at both ends of the visible spectrum. An example of an application of this technique is given in the appendix.

APPENDIX
TEMPERATURE MEASUREMENT BY SCHLIEREN

The technique of crossing a schlieren system with a spectrograph will be illustrated with an example.

Figures 6 and 7 are spectrograms, taken with a prism spectrograph, of a cross section through the center of a ±0.001 radian standard schliere, and of a cross section of the heated gas column five centimeters above the base of a candle flame, just above the tip of the luminous zone. The spectrum of a mercury arc is superimposed in each picture for wavelength identification and to provide a reference base for measurements, since the spectral lines are curved. The schlieren system used was of the familiar parallel beam two-mirror off-set type. The mirror focal lengths were 1.5 meters. The schlieren light source was a ribbon filament lamp. The source slits were 25 microns wide and 1.75 mm high. The source images were made stigmatic by the crossed slit technique described earlier in this report and were magnified eight times with a 50-mm-focal-length camera lens. The wedge interference filter had a reciprocal dispersion of 55 Angstroms per mm and a half-width of 100 Angstroms. The prism spectrograph was assembled from miscellaneous optical components. A system of mirrors was used to rotate the schlieren image 90 degrees on the vertical spectrograph slit. Within wide limits the width of the spectrograph slit affects only the height of the cross section of the object field from which light can enter the spectrograph. In this case, the magnification of the object field on the spectrograph slit was 0.1 and the slit width used was 150 microns, so that the height of the cross section was 1.5 mm.

The wavelength spread in the spectrogram of the heated gas can be seen to be only about 500 Angstroms; it would be desirable to extend the spread over at least the visible spectrum. It is apparent that the sensitivity and precision of the measurements could be improved by such modifications to the apparatus and technique as the following: longer focal length schlieren mirrors, a brighter white light source to permit a narrower source slit width, greater magnification of the source image deflections, a larger numerical value of the reciprocal dispersion of the wedge interference filter and a much narrower half-width (if technically feasible).

If the dispersion of the wedge interference filter is linear with respect to wavelength, and if account is taken of the curvature of the spectral lines and the nonlinear dispersion of the prism spectrograph, it can be seen that a spectrogram, such as figure 7, gives directly a plot of the light deviation angle ϵ vs. distance across the disturbance.

The ϵ values were measured in figure 7 and five other spectrograms of the same disturbance, and the results averaged. Figure 6 was used as a calibration reference. Values of the refractive index variation as a function of the radius of the cylindrical disturbance were calculated by the method of ref. 13 and are plotted in figure 8. From these data, point by point temperature values can be derived by standard methods (see, for example, ref 17). In order to avoid giving a misleading impression of accuracy, the temperatures are not included in this report.

This is a report on developments in the schlieren technique and not on temperature measurements of flames and hot gases. Nevertheless, it is of interest to list the advantages and limitations of the method of temperature measurement which has just been described.

The advantages include:

(1) The phenomenon under investigation is not disturbed by the measurement.

(2) No standard or background source is required.

(3) The method, when it can be applied, gives the gas kinetic temperature, so that equilibrium considerations are avoided.

(4) No quantitative measurements of intensity are required.

(5) The evaluation equation (Eq. 4 in ref 13) is dimensionless, so that the distances that the light rays travel through the disturbance need not be measured and the boundary of the disturbance does not have to be established accurately.

(6) The point by point temperature distribution is found, not an average value through the disturbance.

(7) Diffraction errors, which plague many schlieren measurements, are avoided.

(8) The spectrogram presents the data in such a form that evaluation is direct and simple.

(9) Except that an improved wedge interference filter needs to be developed, the apparatus requirements are not excessive.

The limitations of the method include:

(1) Knowledge of the gas composition is required. Fortunately, in a great many cases of practical interest, the change in refractive index is primarily due to the change in temperature and the effect of change in composition is relatively small. In such cases, if the gas composition is fairly well known, this limitation is not serious.

(2) When the known reference temperature outside the disturbance is low and the temperatures to be measured are high, the process of computing temperatures from refractive index changes requires that a constant be divided by the difference between two numbers of comparable magnitude, a fundamentally inaccurate procedure. The accuracy would be greatly improved if the highest temperature in the disturbance were known rather than the lowest, which unfortunately is rarely the case. In partial compensation for this limitation, the relative temperature distribution within the disturbance is found, even though the scale may be off. It may also be remarked that this limitation is frequently encountered in other high temperature measurement techniques.

(3) At least a reasonably close approximation to axial symmetry is required. If practicable, a considerable number of spectrograms of the same disturbance should be measured and the results averaged. Actually, the requirement of axial symmetry is an advantage. When the schlieren technique is applied to the measurement of temperature by simulating a two-dimensional disturbance, it is necessary to locate the boundary and measure distances accurately within the disturbance, and in some manner to account for end effects. These additional sources of error are avoided with the present method.

In evaluating a schlieren spectrogram such as figure 7 by the method of reference 13, the maximum value of ϵ should be chosen as one of the numbers that is entered into the evaluation table. The distance between this point and the center of the disturbance is then divided into a number of equal intervals, and this interval length is held constant out past the boundary of the disturbance. It is easily seen that the precise location of the boundary is quite immaterial.

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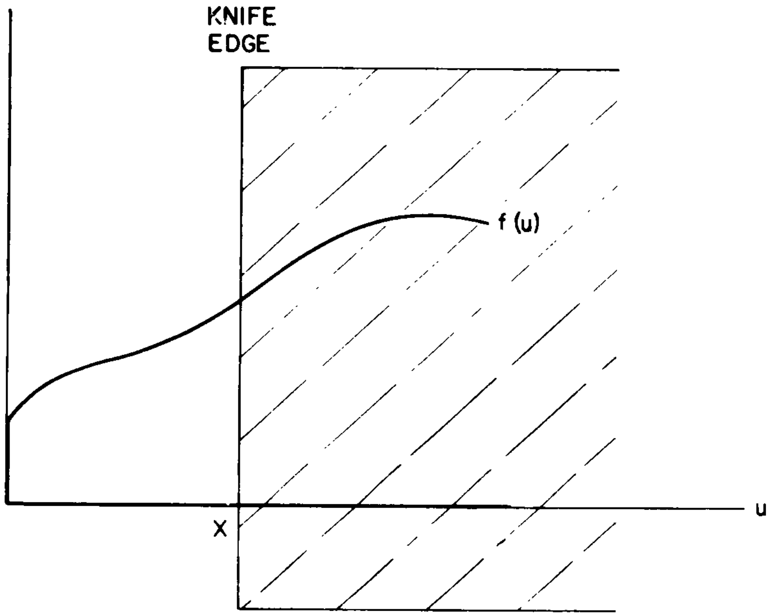

Figure 1. Schlieren Light-Source Image.

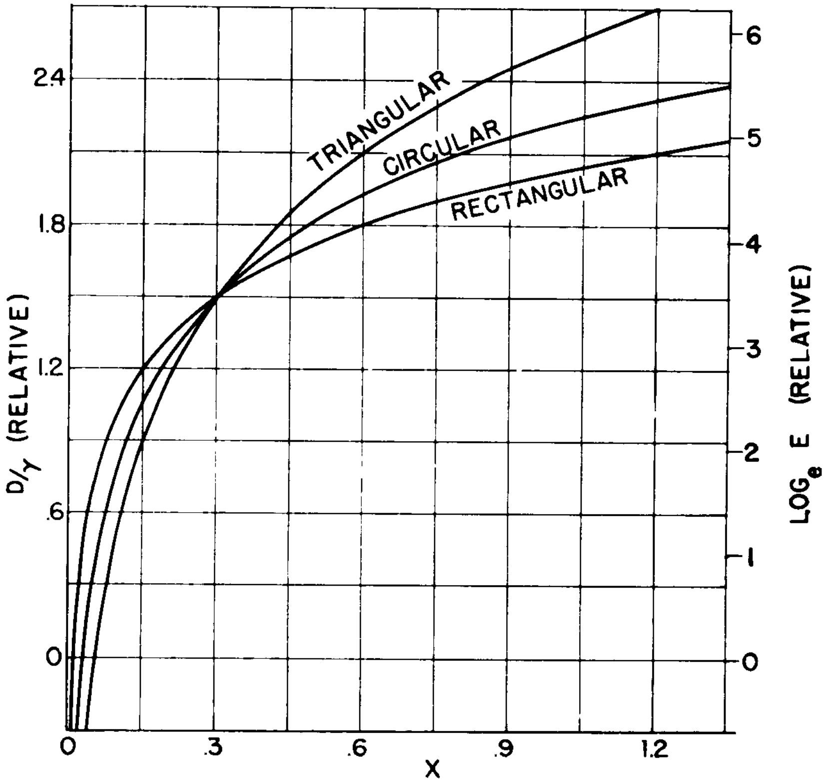

Figure 2. Log E and D/γ vs Knife-Edge Position x for Triangular, Circular, and Rectangular Light-Source Contours.

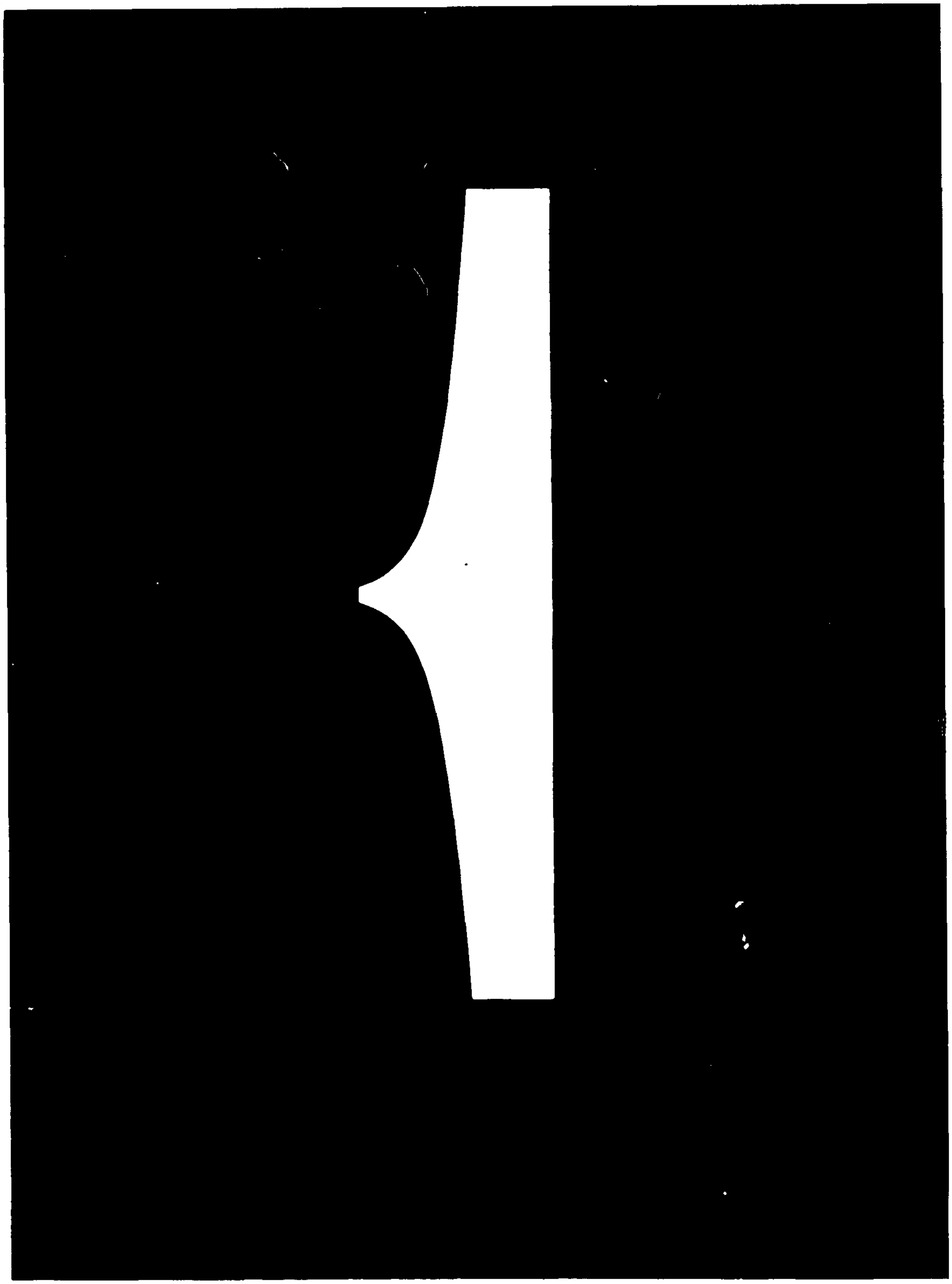

Figure 3. Exponential Source Slit.

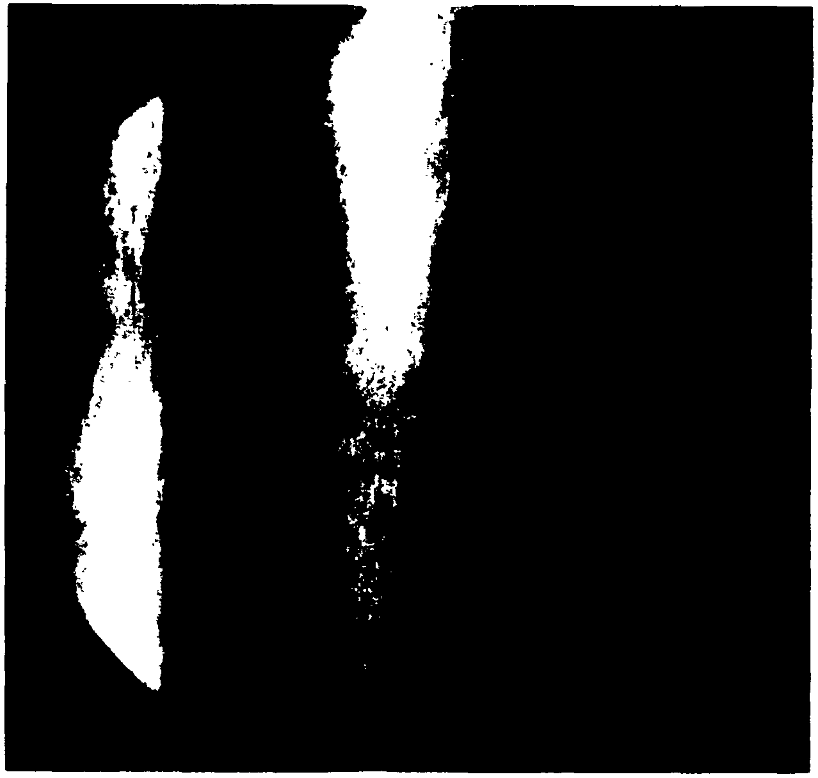

Figure 4. Schlieren Picture of Poor Window. Rectangular Source Slit. S = 5 at Center of Field.

Figure 5. Schlieren Picture with Same Poor Window as in Figure 4. Exponential Source Slit with Constant S = 5.

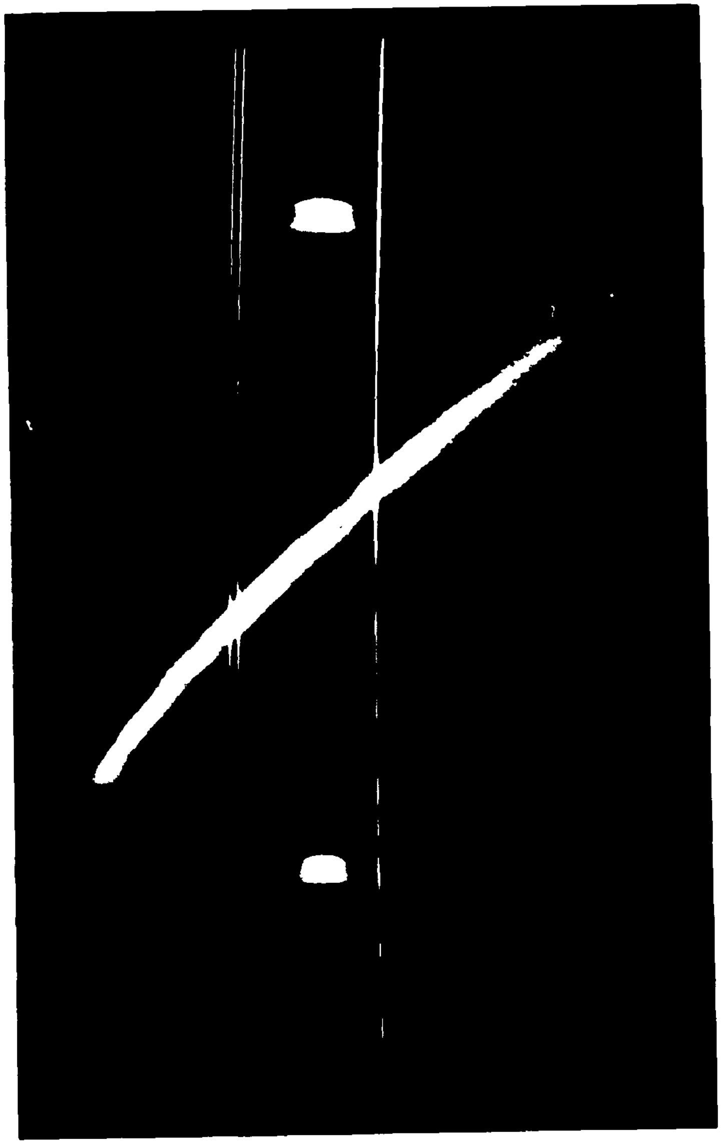

Figure 6. Schlieren Spectrogram of ± 0.001 Radian Standard Schliere.

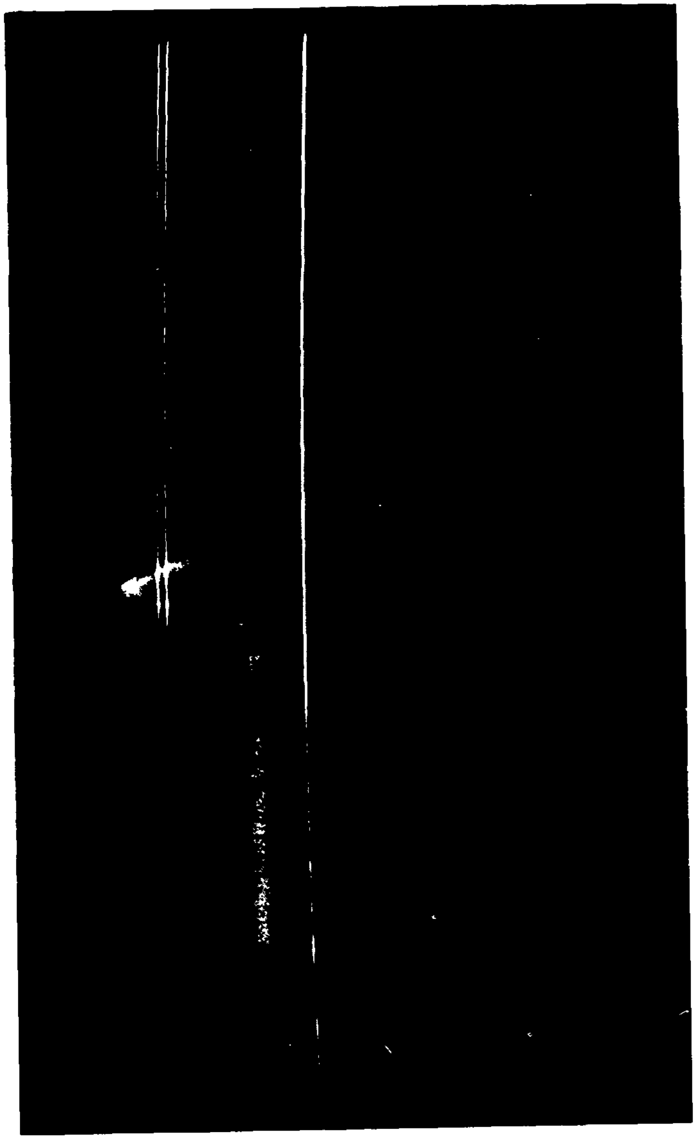

Figure 7. Schlieren Spectrogram of Cross Section of Gas Column 5 cm Above Base of Candle Flame.

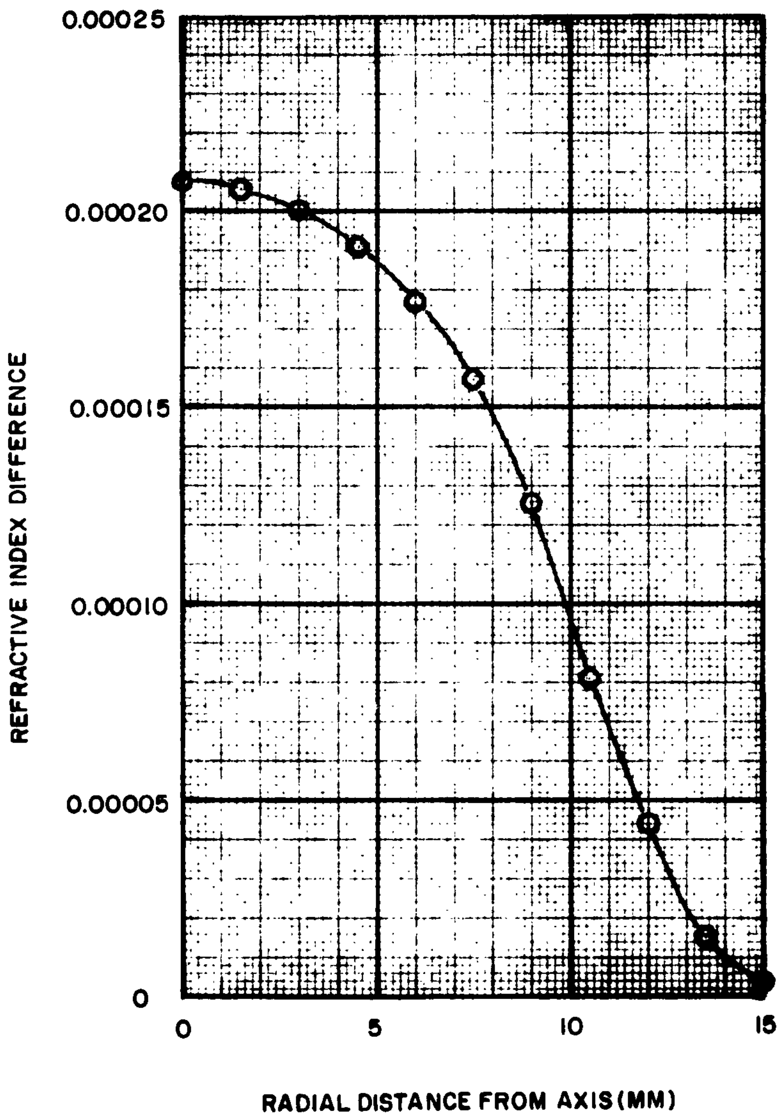

Figure 8. Refractive Index Differences, Relative to Value at Boundary, for Cross Section of Gas Column 5 cm Above Base of Candle Flame. Temperature at Boundary = 300°K.

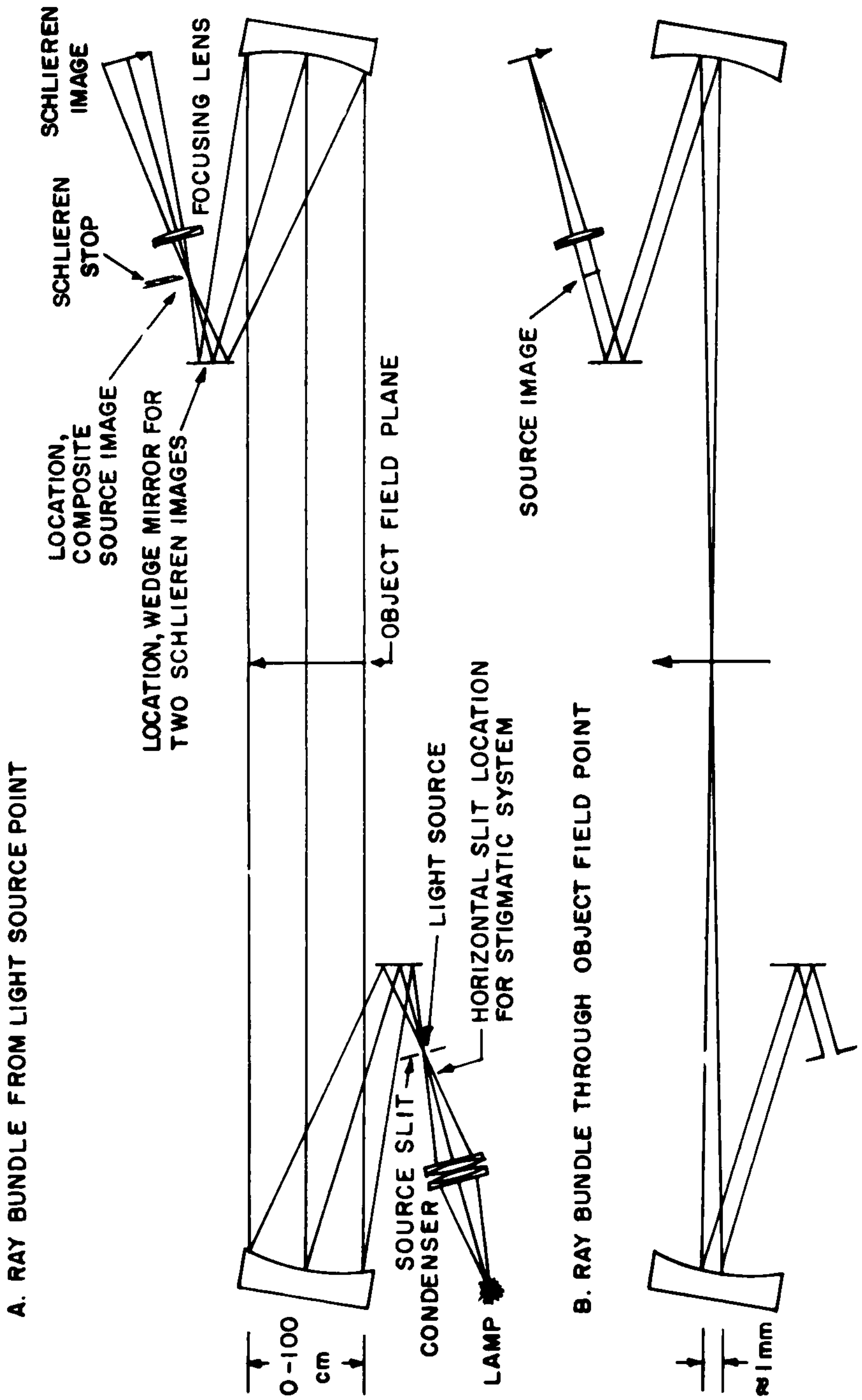

Figure 9. Two-Mirror Offset Schlieren System.

Aeronautical Systems Division, Dir/Engineering Test, Aerodynamics Division, Wright-Patterson AFB, Ohio. Rpt No. ASD-TDR-62-924. A CONTRIBUTION TO THE THEORY OF SCHLIEREN SENSITIVITY AND QUANTITATIVE EVALUATION. Final report, Dec 62, 33p. incl illus., 17 refs.

Unclassified Report

Detectors used to evaluate schlieren images have a response proportional either to the illumination or to the logarithm of the illumination. Sensitivity point functions appropriate to both kinds of response are defined analytically. These functions are well defined even in the presence of optical system imperfections and are not referenced to the clear field illumination. By the choice of an appropriate system configuration, either function can be reduced to a constant. It is shown that, in principle, the light deviation values required for quantitative

(over)

evaluation can be found by subtracting out the illumination increments due to diffraction and other system errors.

A qualitative treatment of schlieren diffraction from the point of view of the Thomas Young theory is given. A technique is described for obtaining a stigmatic source image with an offset two-mirror schlieren system. A simple method for obtaining simultaneous vertical and horizontal knife-edge schlieren pictures is given. A means for magnifying sensitivity without the use of long focal length mirrors is described. A new method for measuring gas temperatures, which involves crossing a color schlieren system with a spectrograph, is described and illustrated with an example.

1. Schlieren Method
2. Temperature Measurement
3. Spectrographic Analysis

I. AFSC Proj 1426, Task 142610
II. Kean, Lee
III. Aval fr OTS
IV. In ASTIA collection

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